Determine if each statement is true or false.

1. The derivative of a sum of two functions is equal to the sum of their derivatives.

   (a) True (Sum rule)
   (b) False

2. The derivative of a product of two functions is equal to the product of their derivatives.

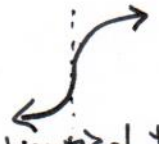
   (a) True
   (b) False $\frac{d}{dx}(fg) = f'g + fg' \neq (f'g')$

3. The value of e may be found using the limit

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n.$$

   (a) True (see page 222)
   (b) False

4. If $f(x)$ is not differentiable at a, then it is not continuous at a.

   (a) True
   (b) False Could have  or 
   Corner or cusp Vertical tangent

5. If $f'(x) = g'(x)$ for every x, then $f(x) = g(x)$ for every x.

   (a) True
   (b) False For example, let $f(x) = x^2$ and $g(x) = x^2 + 1$

6. If $f(x)$ is a polynomial of degree k, then the higher derivative $f^{k+1}(x)$ is zero.

   (a) True
   (b) False

9. Let $f(x) = \frac{100x+x^2+x^3}{x}$. Find a value a such that $f'(a) = 5$.

Simplify first:

$f(x) = 100 + x + x^2$

$f'(x) = 1 + 2x \quad f'(a) = 1 + 2a = 5$

$2a = 4$

$a = 2$

(a) -3
(b) -2
(c) -1
(d) 0
(e) 1
(f) 2
(g) 3
(h) 6
(i) none of the above

10. Let $f(x) = e^{\frac{x}{k}}$. For what value of k does $f''(0) = \frac{1}{4}$?

(a) -3
(b) -2
(c) -1
(d) 0
(e) 1
(f) 2
(g) 3
(h) 6
(i) none of the above

both work: $f(x) = e^{\frac{x}{2}} \Rightarrow f'(x) = \frac{1}{2}e^{\frac{x}{2}}$

$f(x) = e^{-\frac{x}{2}} \Rightarrow f'(x) = -\frac{1}{2}e^{-\frac{x}{2}}$

$\rightarrow f''(x) = \frac{1}{4}e^{\frac{x}{2}}$

13. Let $f(x) = \ln(\ln(\ln(x)))$. What is $f'(x)$?

(a) $\frac{1}{x \ln(x) \ln(\ln(x))}$

(b) $\frac{1}{\ln(\ln(x))}$

(c) the sum of the answers (a) and (b) above

(d) $\frac{1}{x^3}$

(e) none of the above

By the chain rule: $f'(x) = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} =$

14. Consider the circle given by $x^2 + y^2 = 1$. Find the equation for the line tangent to the circle at $P(\frac{3}{5}, -\frac{4}{5})$.

(a) $y + \frac{4}{5} = -\frac{3}{4}(x - \frac{3}{5})$

(b) $y - \frac{4}{5} = -\frac{3}{5}(x - \frac{3}{4})$

(c) $y + \frac{4}{5} = \frac{3}{5}(x - \frac{3}{4})$

(d) $y + \frac{4}{5} = \frac{3}{4}(x - \frac{3}{5})$

(e) $y + \frac{3}{5} = \frac{3}{4}(x - \frac{4}{5})$

(f) none of the above

Implicit differentiation:

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{y}$ $\left.\frac{dy}{dx}\right|_{x=\frac{3}{5}} = -\frac{\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$

$y + \frac{4}{5} = \frac{3}{4}(x - \frac{3}{5})$

17. Let $f(x) = x^x$. Then the derivative $f'(x)$ is

(a) xx^{x-1}

(b) $\ln(x)x^x$

(c) the sum of the answers (a) and (b) above

(d) none of the above

$y = x^x$

$\ln y = \ln(x^x)$

$\ln y = x \ln x$

$\frac{1}{y} y' = \ln x + x \cdot \frac{1}{x}$

$\rightarrow y' = y(\ln x + 1)$

$y' = x^x \cdot \ln x + x^x$

↓ Check to see if c is true

$y' = \ln x \, x^x + x \cdot x^{x-1}$

18. Let $f(x) = e^2$. Then the derivative $f'(x)$ is

(a) e^2

(b) $2e$

(c) the sum of the answers (a) and (b) above

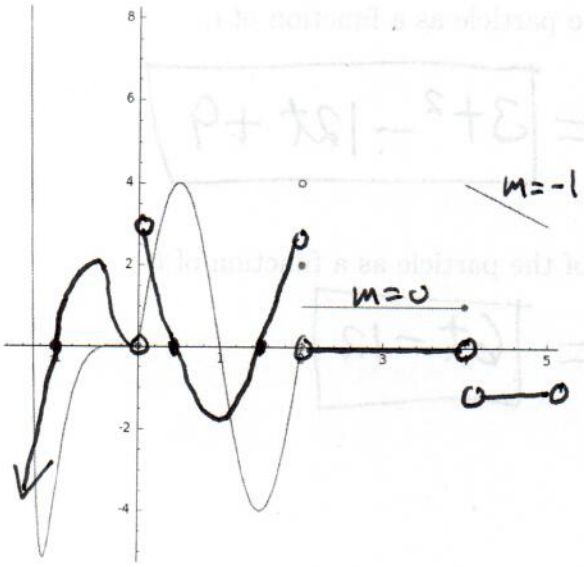
(d) 0

(e) none of the above

e^2 = constant, so the derivative is 0.

Written Problem. Clearly show all steps to receive full credit.

20. Use the given graph of $f(x)$ to answer the questions below.



(a) On the same axes as the graph, sketch the derivative of the given function.

(b) Give an x value where the graph is continuous but not differentiable. Briefly explain why f is not differentiable.

at 0, f(x) is continuous but there is a corner and it is not differentiable.

(c) List the intervals where the function is increasing.

$(-1, 0), (0, \frac{1}{2})$.

(d) List the intervals where the derivative of the function is negative.

$(-\infty, -1), (\frac{1}{2}, \frac{3}{2}), (4, \infty)$

or some reasonable bound

Written Problem. Clearly show all steps to receive full credit.

22. Find the derivative $\frac{dy}{dx}$, using whatever technique is appropriate.

(a) $y = x^{\sin(x)}$

Logarithmic Differentiation

$$\ln y = \ln\left(x^{\sin x}\right)$$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y} y' = \cos x \ln x + \frac{\sin x}{x}$$

$$y' = y\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

$$\boxed{y' = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)}$$

(b) $x^2 + y^2 = xy + 1$

Implicit Differentiation

$$2x + 2y\,y' = y + xy'$$

$$y'(2y - x) = y - 2x$$

$$\boxed{y' = \frac{y - 2x}{2y - x}}$$

(c) $y = \sqrt{\frac{(x^2+4x+12)^3}{(x^3+1)^4}}$

$$\ln y = \ln\left(\sqrt{\frac{(x^2+4x+12)^3}{(x^3+1)^4}}\right) = \frac{3}{2}\ln(x^2+4x+12) - \frac{4}{2}\ln(x^3+1)$$

$$\frac{1}{y} y' = \frac{3}{2}\frac{2x+4}{x^2+4x+12} - 2\frac{3x^2}{x^3+1}$$

$$\boxed{y' = y\cdot\left(\frac{3x+6}{x^2+4x+12} - \frac{6x}{x^3+1}\right)}$$